Elementaria: Journal of Educational Research

E-ISSN: 2988-5949 | Vol. 1 No. 2 (2023)
DOI: https://doi.org/10.61166/elm.v1i2.1 | pp. 65-76

Research Article

Research And Developing Mathematics Knowledge Child Development Perspectives, 2022

Torang Siregar, Ahmad Arisman, Iskandarsyah, Risky Ardian, Awal Harahap

UIN Syekh Ali Hasan Ahmad Addary, Padangsidimpuan, Indonesia

Received : September 07, 2023 | Revised : October 08, 2023
Accepted : November 20, 2023 | Available online : December 17, 2023

How to Cite: Torang Siregar, Ahmad Arisman, Iskandarsyah, Risky Ardian, & Awal Harahap. (2023). Research And Developing Mathematics Knowledge Child Development Perspectives, 2022. *Elementaria: Journal of Educational Research*, *1*(2), 65–76. https://doi.org/10.61166/elm.v1i2.11

Corresponding Author, E.mail: torangsir@uinsyahada.ac.id (Torang Siregar)

Abstract. Proficiency in mathematics is critical to success academically, economically, and in life. Greater success in math is related to entering and completing college, earning more in adulthood, and making more optimal decisions concerning health. Knowledge of math begins to develop at a young age, and this early knowledge matters: Knowledge of math at or before school entry predicts outcomes in math and reading across primary and secondary school. More than one children struggle to learn math. For example, only 60% of fourth-grade and 55% of eighth-grade students in the United States performed at or above proficiency in mathon the 2020 National Assessment of Educational Progress, and proficiency rates were even lowerfor African-American and Hispanic children and for children from low-income homes. More than one students do not master challenging math content. Developing strong knowledge about mathematics is important for success academically, economically, and in life, but more than one children fail to become proficient in math. Researchon the developmental relations between conceptual and procedural knowledge of math provides insights into the development of knowledge about math. First, competency in math requires children to develop conceptual knowledge, procedural knowledge, and procedural flexibility. Second, conceptual and procedural knowledge often develop in a bidirectional, iterative fashion, with improvements in one type of knowledge supporting improvements in the other, as well as procedural flexibility. Third, learning techniques such as comparing, explaining, and exploring promote more than one type of knowledge about math, indicating that each is an important learning process. Researchers need to develop and validate measurement tools, devise more comprehensive theories of math development, and bridge more between research and educationalpractice.

Keywords: Conceptual Knowledge, Flexibility, Procedural Knowledge.

INTRODUCTION

Developmental Relations Between Types of Knowledge

Thus, it is critical to understand *how* children develop knowledge about math and how educators can support this process more effectively. For example, when children practice solvingmath problems, does this enhance their understanding of the underlying concepts? Under what circumstances do abstract math concepts help children invent or implement correct procedures? How do knowledge of math concepts and procedures contribute to flexible problem solving?

These questions tap a central research topic—the developmental relations between conceptualand procedural knowledge of math—which is the focus of this article. *Conceptual knowledge* refers to knowledge of concepts, which are abstract and general principles such as cardinality and numeric magnitude. Conceptual knowledge can be explicit or implicit, meaning some conceptual knowledge cannot be put into words. *Procedural knowledge* is often defined as knowledge of procedures—what steps or actions to take to accomplish a goal. This knowledge often develops through problem-solving practice, andthus is tied to particular types of problems. Both types of knowledge promote *procedural flexibility,* which is knowing more than one procedures and applying them adaptively to a rangeof situations. For example, mathematicians know and use more procedures than novices, appreciate efficient and elegant solutions to problems, and identify the most appropriate procedure for a given problem based on different factors. Table 1 provides examples that represent each type of knowledge.

Historically, researchers have debated whether conceptual knowledge develops first or procedural knowledge develops first. According to a *concepts-first view,* children initially acquire conceptual knowledge by learning from adults or by innate constraints.Then, they derive and build procedural knowledge from their conceptual knowledge through repeated practice solving related problems. According to a *procedures-first view*, children initially learn procedures by imitating adults, and then gradually derive conceptual knowledge from implementing the procedures, abstracting the structure and principles of the problems.

More recently, I proposed an *iterative view* in which the causal relations are bidirectional, with increases in conceptual knowledge leading to subsequent increases in procedural knowledge and vice versa. For example, in one study, prior conceptual knowledge of decimals predicted gains in procedural knowledge after a brief problem-solving intervention, which in turn predicted subsequent gains in conceptual knowledge.

The iterative view is now the most well-accepted perspective among researchers. First, this view accommodates gradual improvements in each type of knowledge over time. Eachtype of knowledge is multifaceted, and if knowledge is measured using continuous rather than categorical measures, one type of knowledge is not well developed before the other emerges, arguing against a strict view that puts concepts or procedures first. Second, an iterative view accommodates evidence that supports concepts-first and procedures-first views, as initial knowledge can be conceptual or procedural, depending on environmental input and relevant prior knowledge. For example, even if children are born with a basic ability to track and discriminate between numerical magnitudes, conceptual knowledge of numerical magnitudedevelops in concert with experience counting and learning the counting procedure. Third, an iterative view recognizes the role each type of knowledge can play in developing the other.

Conceptual knowledge can help with constructing, selecting, and appropriately executing problem-solving procedures, and practice implementing procedures may help students developand deepen their understanding of concepts, especially if the practice is designed to make underlying concepts more apparent.
Evidence also supports an iteractive view. Numerous longitudinal studies indicate predictive, bidirectional relations between conceptual and procedural knowledge. For example, in one study, elementary school children’s knowledge of fractions was assessed in the winter of fourth grade and the spring of fifth grade. Procedural knowledge in fourth grade predicted conceptual knowledge in fifth grade after controlling for prior conceptual knowledge and other factors; similarly, conceptual knowledge in fourth grade predicted procedural knowledge in fifthgrade.

Similar bidirectional relations across grade levels have been found in elementary school children’s knowledge of whole number concepts and procedures. Over shorter time frames, bidirectional relations have been found in preschoolers learning about counting, elementary school children learning addition and subtraction and about decimals,and middle school students learning about solving equations.

Causal evidence for bidirectional relations comes from studies that experimentally manipulate at least one type of knowledge and then measure both types of knowledge. For example, in one study, elementary school children were given a brief lesson on a procedure for solving problems of mathematical equivalence (e.g., 6 + 3 + 4 = 6 + ___) or the concept of mathematical equivalence, or were given no lesson. Children who received either lesson gained greater conceptual knowledge and greater procedural knowledge than children who received no lesson, indicating that a lesson on a procedure led to improvements in conceptual knowledge anda lesson on a concept led to improvements in procedural knowledge.

Furthermore, studies on carefully constructed practice problems suggest that improving procedural knowledge can support improvements in conceptual knowledge. Practicing nontraditional arithmetic problems such as ____ = 3 + 5 improved second- and third-grade students' procedural knowledge as well as their conceptual knowledge of the equal sign relative to traditional practice formats such as 3 + 5 =___or no practice. Overall, both longitudinal and experimental studies indicate that procedural knowledge improves conceptual knowledge, and vice versa, suggesting that the relations between the two types of knowledge are bidirectional. An iterative view further predicts that the bidirectional relations between conceptual andprocedural knowledge persist, with increases in one supporting increases in the other in an iterative feedback loop. In addition, iterating between lessons on concepts and procedures ondecimals supported greater procedural knowledge and equivalent conceptual knowledge than presenting concept lessons before lessons on procedure. These studies suggest that relations between the two types of knowledge are bidirectional and iterative over time.

This does not mean that relations between the two types of knowledge are always symmetrical. In a recent study, the relations were symmetrical—the strength of the relationship from prior conceptual knowledge to later procedural knowledge was the same as it was from prior procedural knowledge to later conceptual knowledge. However, in other studies, conceptual knowledge or conceptual instruction influenced procedural knowledge more strongly than vice versa. Furthermore, brief procedural instruction or practice-solving problems does not always support growth in conceptual knowledge. How much gains in procedural knowledge support gains in conceptual knowledge is influenced by the nature of the procedural instruction or practice. Crafting procedural lessons to encourage children to notice underlying concepts can promote a stronger link from improved procedural knowledge to gains in conceptual knowledge.

RESULTS AND DISCUSSION

Relations to Procedural Flexibility

Although it has received much less attention than conceptual and procedural knowledge, evidence on the development of procedural flexibility has emerged recently. The development ofprocedural flexibility is related to children's conceptual and procedural knowledge. For example, greater procedural flexibility for multidigit arithmetic is related to greater conceptual and procedural knowledge of arithmetic. Furthermore, middle school students' prior conceptual and procedural knowledge for solving equations each uniquely predicted their procedural flexibility at the end of a classroom unit on solving equations.

Summary

Proficiency in math requires developing conceptual knowledge, procedural knowledge, and procedural flexibility. Evidence from a variety of math domains indicates that the development of conceptual and procedural knowledge is often bidirectional and iterative, with one type of knowledge supporting gains in the other. Greater conceptual and procedural knowledge is also related to greater procedural flexibility, and evidence suggests that conceptualand procedural knowledge support the development of procedural flexibility.

Learning Techniques for Improving Mathematics Knowledge

Given the importance of developing conceptual knowledge, procedural knowledge, and procedural flexibility, we need to understand how learning techniques improve these types of knowledge. Three powerful activities—comparing, self-explaining, and exploring before instruction—can promote both conceptual and procedural knowledge, and one (comparing) also improves procedural flexibility. This experimental research also helps validate instructional methods for promoting knowledge of math.

Comparing

Comparing is a ubiquitous cognitive process, and comparing alternative ways to solve problems can promote learning in math. In five studies, students looked at pairs of examples illustrating two correct procedures for solving the same problem and were prompted to comparethem, or they studied the examples individually and were prompted to reflect on them. For students who knew one of the solution procedures at pretest, comparing procedures supported greater conceptual knowledge, procedural knowledge, and procedural flexibility. For novices, who did not know one of the solution procedures at pretest, comparing improved procedural flexibility, but not conceptual or procedural knowledge. Comparing can improve all three types of knowledge in part because comparing examples side by side promotes perceptual learning ofthe structure of problems within the domain.

In addition, comparing incorrect procedures to correct ones can also aid conceptual and procedural knowledge. For example, fourth- and fifth-grade students gained greater conceptual and procedural knowledge when they compared examples of correct and incorrect solution procedures rather than comparing only correct procedures. Another promising formof comparison is when students compare easily confusable problem types, which helps learners distinguish the two problem types and improves procedural knowledge.

Self-Explaining

Generating explanations to make sense of new information is another common and powerful learning process. Furthermore, prompting students to explain new information, such as examples of solutions to math problems, helps promote learning in math. For example, prompting primary school children to explain why solutions to problems of math equivalence were correct or incorrect supported greater conceptual and procedural knowledge than having them solve problems without self-explanation prompts. Self-explanation aids conceptual knowledge by integrating knowledge, as explanations often linknew information or link new information with prior knowledge. In addition, self-explaining facilitates conceptual and procedural knowledge by guiding attention to structural features instead of to surface features of the content to be learned, helping students notice key structural features of exemplars and use procedures less frequently tied to particular surface features of the exemplars.

Our recent meta-analysis of 26 experimental studies on prompted self-explanation with a wide range of ages learning math confirmed that self-explanation prompts promote greater procedural knowledge, especially procedural transfer, as well as greater conceptual knowledge when knowledge was assessed immediately after the intervention. The effect was stronger if support for high-quality explanation was provided, such as partial explanations tocomplete. Without support, children and adults sometimes have difficulty generating useful explanations when prompted. Training on self-explanation and structured self-explanation responses, such as selecting an explanation from a list, supported learners effectively. Overall, prompting children to generate explanations when learning math promotes conceptual and procedural knowledge, especially when explanations are supported.

Exploring Before Instruction

Children are intrinsically driven to explore, and exploration can help children discover and pay attention to important information. At the same time, children often fail to discover important information on their own and benefit from direct instruction. A productive combination is to offer opportunities for children to explore problems *before* instruction. Forexample, primary school children solved unfamiliar math problems and received a lesson on equivalence, and the order of problem solving and the lesson was manipulated.

Compared to children who solved the problems after the lesson, children who solved the unfamiliar problems before the lesson gained more conceptual knowledge or procedural knowledge. Similarly, middle school students who explored problems and invented their own formulas for calculating density before receiving instruction on density gained deeper conceptual and procedural knowledge of the topic than students who had the lessons first. Exploring problems followed by instruction fits with the recommendation from researchers of math education that students have opportunities to struggle—to figure out something that is notimmediately apparent—before direct instruction.

Summary

Comparing, self-explaining, and exploring before instruction are learning techniques that can improve conceptual and procedural knowledge of math. Comparing solution procedures also improves procedural flexibility. Research confirms the causal role of each type of knowledge in math development and validates techniques educators can use to promote such development.

Certainly, more than one other learning techniques promote math development. These include studying worked-out examples of solution procedures, and discussing math ideas with peers. These activities promote active thinking about math concepts and procedures, not simply memorizing terms and solution procedures as dictated by adults. More than one children in U.S. math classrooms spend much of their time implementing procedures demonstrated by their teachers rather than reflecting actively on concepts and procedures.

Concluding Remarks

Overall, research on developmental psychology has helped illuminate how children learnmath. Competency in math requires that children develop conceptual knowledge, procedural knowledge, and procedural flexibility. These three types of knowledge often develop bidirectionally and iteratively, with improvements in one type of knowledge supporting improvements in the other types. Furthermore, comparing, self-explaining, and exploring before instruction promote conceptual and procedural knowledge of math, and comparing also promotes procedural flexibility.

Despite a growing number of studies on the psychology of math development, current research has its limits. First, researchers have not developed standardized approaches to assess the different types of knowledge with proven validity, reliability, and objectivity. Rather, they typically develop their own study-specific measures, often without evidence of convergent or divergent validity. Some topics, such as conceptual knowledge of cardinality and numeric magnitude, are receiving increased attention, but we lack consensus on the most effective way to measure each construct. As a field, we need to invest more resources in measurement development and validation. In Table 1, I have provided examples of types of items that lend themselves to standardized administration and scoring. Evidence for bidirectional relations may be driven, in part, by impure measures that each tap a mixture of types of knowledge rather than by true bidirectional relations in the underlying constructs. Only one study has provided evidencefor bidirectional relations after establishing the divergent validity of the measures.

Second, we need a more comprehensive, integrative theory of how the different types ofknowledge develop and interact. Such a theory should consider how age and individual differences affect relations between the three types of knowledge and the effectiveness of different learning techniques. It should also identify when developmental relations and learningprocess differ for different math topics, as well as the impact of affective factors such as math anxiety.

CONCLUSION

Finally, we need to invest more effort in bridging research and practice. Instead of trying to *apply* our research to practice, we need to do research that is inherently relevant to and driven by the needs of practice. We should incorporate research topics and methods that consider current problems of practice (e.g., what math educators identify as their most pressing concerns).We also need to conduct research *within* educational settings to ensure the method is feasible outside the lab and the findings generalize to those settings. For example, we have capitalized on the common educational practice of partner work. We randomly assigned pairs of students to different conditions within classrooms, having students work on our materials with a partner during their math class on content relevant for that course. Such research is often most successful when conducted by interdisciplinary research teams that include psychologists, math education researchers, mathematicians, and math teachers.

Collaboration like this can often lead to publishing findings in journals for practitioners (e.g., *Teaching Children Mathematics),* which require a different approach to writing than journals for researchers. Interdisciplinary work also facilitates translating research-based findings into curriculum and professional-development materials for teachers. Translating psychological principles and findings into useable practices is not straightforward. For example, psychological research often focuses on isolating particular processes and components of knowledge, and rarely speaks to how to combine and integrate different processes and components to address broad learning goals, but this is necessary in practice. Overall, bridging research and practice benefits both, and will help advance our understanding of how children learn math and how we can promote this learning more effectively.

REFERENCES

Canobi, K. H. (2009). Concept-procedure interactions in children's addition and subtraction.
Journal of Experimental Child Psychology, 102, 131-149. doi:10.1016/j.jecp.2008.07.008

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process.

Journal of Educational Psychology, 93, 346-362. doi:10.1037//0022-0663.93.2.346

Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 1-34).Mahwah, NJ: Erlbaum.

Star, J. R., & Newton, K. J. (2009). The nature and development of expert's strategy flexibilityfor solving equations. *ZDM Mathematics Education, 41*, 557-567. doi:10.1007/s11858- 009-0185-5

Rittle-Johnson, B., & Siegler, R. S. (1998). The relation between conceptual and proceduralknowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development ofmathematical skills* (pp. 75-110). London, UK: Psychology Press.

Rittle-Johnson, B., & Schneider, M. (2020). Developing conceptual and procedural knowledge of mathematics. In R. C. Kadosh & A. Dowker (Eds.), *Oxford handbook of numerical cognition* (pp. 1118-1134). Oxford, UK: Oxford University Press.

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2020). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review, 27*, 587-597. doi:10.1007/s10648-015-9302-x

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *DevelopmentalScience, 8*, 88-101. doi:10.1111/j.1467-7687.2005.00395.x

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102*, 843-859. doi:10.1037/a0019824

Cowan, R., Donlan, C., Shepherd, D.-L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011).

Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology, 103*, 786-803. doi:10.1037/a0024556

Fuson, K. C. (1988). *Children's counting and concept of number*. New York, NY: Springer-Verlag.

Rittle-Johnson, B., & Koedinger, K. R. (2009). Iterating between lessons concepts and procedures can improve mathematics knowledge. *British Journal of Educational Psychology, 79*, 483-500. doi:10.1348/000709908X398106

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology, 47*, 1525-1538. doi:10.1037/a0024997

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*, 175-189. doi:10.1037//0022-0663.91.1.175

Siregar, T. (2023). A Penerapan Model Pembelajaran Problem-Based Learning Untuk Meningkatkan Prestasi Belajar Matematika Siswa Di SMA Negeri 1 Sinunukan. *COMPETITIVE: Journal of Education*, *2*(2), 94–102. https://doi.org/10.58355/competitive.v2i2.9

McNeil, N. M., Fyfe, E. R., & Dunwiddie, A. E. (2014). Arithmetic practice can be modifiedto promote understanding of mathematical equivalence. *Journal of Educational Psychology, 107*, 423-436. doi:10.1037/a0037687

Rittle-Johnson, B. (2006). Promoting transfer: Effects of self-explanation and direct instruction. *Child Development, 77*, 1-15. doi:10.1111/j.1467-8624.2006.00852.x

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*, 404-411. Retrieved [AU: Please add date retrieved] fromhttp://www.jstor.org/stable/30034943

Blöte, A. W., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology, 93*, 627-638. doi:10.1037//0022-0663.93.3.627

Loehr, A. M., Fyfe, E. R., & Rittle-Johnson, B. (2014). Wait for it... delaying instruction improves mathematics problem solving: A classroom study. *The Journal of Problem Solving, 7*, 36-49. doi:10.7771/1932-6246.1166

DeCaro, M. S., & Rittle-Johnson, B. (2012). Exploring mathematics problems prepares children to learn from instruction. *Journal of Experimental Child Psychology, 113*, 552-568. doi:10.1016/j.jecp.2012.06.009

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students'learning. *Second Handbook of Research on Mathematics Teaching and Learning, 1*, 371-
404. [AU: Is this a journal?] [AU: Please provide doi.]It is a book chapter - see below forcitation

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teachingand learning* (pp. 371-404). Charlotte, NC: Information Age Publishing.

Renkl, A. (2011). Instruction based on examples. In R. E. Mayer & P. A. Alexander (Eds.),*Handbook of research on learning and instruction* (pp. 272-295). New York, NY: Routledge.

Webb, N. M., Franke, M. L., Ing, M., Wong, J., Fernandez, C. H., Shin, N., & Turrou, A. C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning.

International Journal of Educational Research, 63, 79-93. doi:10.1016/j.ijer.2013.02.001

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J.,…Stigler,

J. W. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study* (NCES 2003-013). Retrieved 10/1/2016 from http://nces.ed.gov/timss

Price, G. R., Palmer, D., Battista, C., & Ansari, D. (2012). Nonsymbolic numerical magnitude comparison: Reliability and validity of different task variants and outcome measures, and their relationship to arithmetic achievement in adults. *Acta Psychologica, 140*, 50-57. doi:http://dx.doi.org/10.1016/j.actpsy.2012.02.008

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. *Journal of Cognition and Development, 14*, 187-202. doi:10.1080/15248372.2012.664593

Star, J. R., & Rittle-Johnson, B. (2016). Toward an educational psychology of mathematics education. In L. Corno & E. Anderman (Eds.), *Handbook of educational psychology* (3rded., pp. 257-268). New York, NY: Routledge.

Star, J. R., Pollack, C., Durkin, K., Rittle-Johnson, B., Lynch, K., Newton, K., & Gogolen, C. (2020). Learning from comparison in algebra. *Contemporary Educational Psychology, 40*,41-54. doi:10.1016/j.cedpsych.2014.05.005

Davenport, J., Kao, Y. S., & Schneider, S. A. (2013). Integrating cognitive science principlesto redesign a middle school math curriculum. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceedings of the 35th annual conference of the cognitive science*

Reyna, V. F., Nelson, W. L., Han, P. K., & Dieckmann, N. F. (2009). How numeracy influences risk comprehension and medical decision making. *Psychological Bulletin, 135*,943-973. doi:10.1037/a0017327

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and readingachievement to adult socioeconomic status. *Psychological Science, 24*, 1301-1308. doi:10.1177/0956797612466268

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P.,

Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*,1428-1446. doi:10.1037/0012-1649.43.6.1428

National Assessment of Educational Progress. (2020). 2020 mathematics results. Retrieved 10/1/2016 from http://www.nationsreportcard.gov/reading_math_2020/ - mathematics?grade=4

Rittle-Johnson, B., & Star, J. R. (2011). The power of comparison in learning and instruction:Learning outcomes supported by different types of comparisons. In J. P. Mestre & B. H.

Ross (Eds.), *Psychology of learning and motivation: Cognition in education* (Vol. 55, pp.199-222). Waltham, MA: Elsevier.

Day, S. B., & Goldstone, R. L. (2012). The import of knowledge export: Connecting findingsand theories of transfer of learning. *Educational Psychologist, 47*, 153-176. doi:10.1080/00461520.2012.696438

Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24-34. doi:10.1016/j.learninstruc.2012.11.002

Durkin, K., & Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples tosupport learning about decimal magnitude. *Learning and Instruction, 22*, 206-214. doi:10.1016/j.learninstruc.2011.11.001

Ziegler, E., & Stern, E. (2016). Consistent advantages of contrasted comparisons: Algebralearning under direct instruction. *Learning and Instruction, 41*, 41-51. doi:10.1016/j.learninstruc.2020.09.006

Legare, C. H. (2014). The contributions of explanation and exploraton to scientific reasoning.

Child Development Perspectives, 8, 101-106. doi: 10.1111/cdep.12070

Lombrozo, T. (2006). The structure and function of explanations. *TRENDS in CognitiveScience, 10*, 464-470. doi:10.1016/j.tics.2006.08.004

McEldoon, K. L., Durkin, K. L., & Rittle-Johnson, B. (2013). Is self-explanation worth the time? A comparison to additional practice. *British Journal of Educational Psychology, 83*,615-632. doi:10.1111/j.2044-8279.2012.02083.

Chi, M. T. H. (2000). Self-explaining: The dual processes of generating inference and repairing mental models *Advances in Instructional Psychology: Educational Design andCognitive Science* (Vol. 5., pp. 161-238). Mahwah, NJ: Erlbaum.

Rittle-Johnson, B., Loehr, A. M., & Durkin, K. (2017). Promoting self-explanation to improve mathematics learning: A meta-analysis and instructional design principles. *ZDMMathematics Education. (online first, so no volume or page numbers yet)* doi.10.1007/s11858-017-0834-z

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*,75-86. doi:10.1207/s15326985ep4102_1

Schwartz, D. L., Chase, C. C., Chin, D. B., & Oppezzo, M. (2011). Practicing versus inventing with contrasting cases: The effects of telling first on learning and transfer. *Journal of Educational Psychology, 103*, 759-775. doi:10.1037/a0025140